CALCULUS.

395. Proposed by W. W. BURTON, Mercer University, Macon, Ga.

Into a full conical wine glass whose depth is a and whose angle at the base is 2α there is carefully dropped a spherical ball of such size as to cause the greatest overflow. Show that the radius of the ball is $a \sin \alpha/(\sin \alpha + \cos 2\alpha)$.

From Woods and Bailey's *A Course in Mathematics* (1907), Volume I, page 213.

396. Proposed by ELBERT H. CLARKE, Purdue University.

The length of the curve $y = x^n$ from the origin to the point (1, 1) is given by the formula

$$l = \int_0^1 \sqrt{1 + n^2x^{2n-2}}dx.$$

Our geometric intuition would tell us that the limit of this length as n becomes infinite is 2. Give a strict analytic proof that

$$\lim_{n\to\infty} \int_0^1 \sqrt{1 + n^2x^{2n-2}}dx = 2.$$

MECHANICS.

315. Proposed by H. S. UHLER, Yale University.

A solid, homogeneous, right, circular cylinder is allowed to move from rest down a circular cylindrical track which is concave upwards. Find the ratio of the radius of the track to the radius of the cylinder when the time of descent through a finite arc to the bottom is the same for the extreme cases of no slipping and zero friction. Show also that the same relation holds for a sphere descending a cylindrical or spherical surface.

316. Proposed by C. N. SCHMALL, New York, N. Y.

A body at rest at a point R begins to move towards a center of force F. The distance $RF = d$, and the force varies inversely as the distance. Two intermediate points in the path are P and Q, such that $FP = kd$, and $FQ = k^nd$. Show that the body will traverse the distance QP in a *maximum* of time if $k = 1/n^{\frac{1}{2(n-1)}}$.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

426. Proposed by HERBERT N. CARLETON, West Newbury, Mass.

Find all solutions of the equation

$$x\sqrt[x]{x} = x^x.$$

Solution by J. A. Capron, Notre Dame, Ind.

The equation may be written in the form $x^{1+(1/x)} = x^x$, or $x^{(x+1)/x} - x^x = 0$. Factoring, we have $x^x[x^{(x+1-x^2)/x} - 1] = 0$. This equation is equivalent to the two equations $x^x = 0$ and $x^{(x+1-x^2)/x} - 1 = 0$. The first of these equations is satisfied for the value of $x = -\infty$. From the second equation, we have, by taking logarithms, the equation

$$\left(\frac{x+1-x^2}{x}\right)\log x = 0.$$

This equation is equivalent to the three equations $1/x = 0$, $x + 1 - x^2 = 0$, and $\log x = 0$. From the first of these equations, $x = \pm\infty$; from the second, $x = (1 \pm \sqrt{5})/2$; and from the third, $x = 1$.